RECONSTRUCTION THEOREMS IN NONCOMMUTATIVE GEOMETRY

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The famed Gel'fand-Naimark duality establishes a categorical equivalence between compact Hausdorff spaces and commutative unital C^*-algebras, motivating the identification of C^*-algebras as noncommutative topological spaces. More recently, Connes proposed identifying noncommutative Riemannian manifolds with spectral triples, where a compact spin manifold X can be identified with the triple $(C(X), L^2(X,S), D)$, for S the spinor bundle and D the Dirac operator on X. Gel'fand-Naimark, at present, then partially generalises in the form of Connes's reconstruction theorem, which gives an abstract characterisation of certain similar triples arising from a compact oriented Riemannian manifold, namely, (orientable) commutative triples. Commutative spectral triples, however, are far from the only interesting class of global-analytically defined spectral triples. Indeed, we obtain a reconstruction theorem for almost-commutative triples, the triples which arise in applications of noncommutative geometry to particle physics and cosmology, by sharpening Connes's result to an abstract characterisation of commutative triples arising from Dirac-type operators on compact oriented Riemannian manifolds. Moreover, we discuss progress towards a reconstruction theorem for another global-analytically defined and physically relevant, but more non-trivially noncommutative class of spectral triples, namely, isospectral deformations.