RECURRENCE PROPERTIES OF DISCRETE TIME UNITARY EVOLUTIONS

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We study the orbit of a fixed initial state ϕ of a quantum system under a discrete time evolution induced by a unitary operator U and discuss its recurrence properties. For classical random walks the probability to return to the initial state eventually is completely characterized by the probabilities to return in exactly n steps. In order to define a notion of first arrival in the quantum mechanical setting we introduce an explicit measurement that determines whether the system has returned to the state ϕ. This measurement is performed after each time step and we call a system recurrent if, with probability one, the system will eventually be found again in its initial state. We show that a system is recurrent iff the spectral measure of ϕ with respect to U as no absolutely continuous part. Moreover for recurrent systems the expected first return time is either an integer or infinite. We give a topological interpretation of this number in terms of the Schur function of the spectral measure.