KINETIC THEORY OF THE HUBBARD MODEL: A BOLTZMANN EQUATION WITH A TWIST

Jani Lukkarinen
Department of Mathematics and Statistics, University of Helsinki

In this joint work with Peng Mei and Herbert Spohn we consider the standard Hubbard model which is a simplified model for the evolution of fermions on a lattice with an on-site quartic interaction. In the kinetic scaling limit, space and time scaled by λ^2 where λ is the strength of the interaction term, we obtain a matrix-valued Boltzmann equation. The Hubbard model conserves energy and the total spin, including its direction, and since the anharmonic interaction is between the spin-components, it will vanish if one of the components vanishes. These properties are reflected in several novel features of the Boltzmann equation: it has an additional effective Hamiltonian term describing a rotation of the spin-basis on the kinetic scale, as well as special "degenerate" solutions which have no collisions and do not equilibrate. Even perturbatively, checking the validity of the Boltzmann equation requires an expansion at all orders in λ, and novel methods need to be developed because the product is no longer commutative. Thus it is important to check that the transport equation is both physically and mathematically sensible. We present several properties of the Boltzmann equation which support such a conclusion.