AN EXPLICIT INNER APPROXIMATION TO THE ONE-BODY QUANTUM MARGINAL POLYTOPE

David Gross, Matthias Christandl, Michael Walter
University of Freiburg, ETH Zürich, ETH Zürich

The *one-body quantum marginal problem* (1-QMP) asks which single-site reduced density matrices arise from globally pure states. We show that this set allows for an explicit inner approximation, which becomes increasingly accurate as the number of particles grows.

The 1-QMP had been a long-standing open problem in quantum many-body theory. Recently, Klyachko recognized the set in question as a *moment polytope* in the sense of symplectic geometry and gave a recipe for computing its facets. However, both the algorithm and the polytope itself seem to be highly non-trivial and only a few cases have been explored. We present first systematic results on the properties of the polytope for large quantum systems.

More precisely: the set of eigenvalues of the one-body reduced density matrices of globally pure states allow for a simple *outer* approximation. It is obtained by imposing only positivity and normalization constraints. We show that for an n-body system, there is an explicit *inner* approximation, whose distance to the outer one is bounded by $(\log n)/n$. For reasons that will become clear (and are by no means connected to the host nation of the present conference), we refer to the inner polytope as a *lego simplex.* There is a sense in which the construction is tight, up to, possibly, the log-factor.

Perhaps surprisingly, the technical analysis rests on an analytic diagonalization of a certain random walk on the representations of the symmetric group.